Pearson Edexcel

Mark Scheme (FINAL)

October 2019

Pearson Edexcel IAL Mathematics (WMA12)

Pure Mathematics P2

Question Number	Scheme	Marks
1 (a)	$y=2x^2(x-5)=2x^3-10x^2$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-20x$	M1
	Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 6x^2-20x=0\Rightarrow x=0,\frac{10}{3}$ oe	dM1 A1
		(4)
(b)	$x\leqslant 0,\quad x\geqslant\frac{10}{3}$	M1 A1
		(2)
		(6 marks)
Alt (a)	$\left(\frac{\mathrm{d(uv)}}{\mathrm{d}x}=uv'+vu'\right)\qquad u=2x^2,u'=4x,v=x-5,v'=1$	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)Ax(x-5)+Bx^2$	M1
	Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 4x\mathbf{(}x-5\mathbf{)}+2x^2=0\Rightarrow x=0,\frac{10}{3}$	dM1 A1

(a)

B1 Expands to a correct form $(y=)\ 2x^2(x-5)=2x^3-10x^2$

(may be implied by sight of $\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-20x$)

If a candidate uses the product rule then award this mark for correct terms for *u*, *u'*, *v* and *v*' stated or implied.

M1 Differentiated reducing a power by one on one of their terms.

If a candidate uses the product rule then look for $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)Ax(x-5)+Bx^2$. (Do not award for just differentiating the factor in front of the bracket and the $x-5$)

dM1 $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and solves a quadratic equation to find at least one solution. (Usual rules for solving a quadratic). Values such as 0 and 3.33 to imply this mark.

It is dependent on the previous method mark. Dividing by x to find one of the solutions is also acceptable for this mark.

Allow answers to be just written down but you may need to check these on your calculator.

A1 $x = 0, \frac{10}{3}$ oe (Must be seen in (a)

Allow if embedded within pairs of coordinates for the stationary points of C
SC If no calculus work shown then 0001 for achieving the correct x coordinates.

SC If no credit worthy work seen in (a) but they use calculus in (b) then B1M1M1 can be awarded but withhold the A mark.

(b) Note we are now marking this on epen as M1A1

M1 One of $x \le "0"$ or $x \ge "\frac{10}{3}"$. Allow for $x < "0"$ or $x > "\frac{10}{3}"$.

They must have only achieved a maximum of two x coordinates in (a).
Do not award for inequalities using solutions from $y = 0$ or if they have integrated instead.

A1 Both $x \le 0, x \ge \frac{10}{3}$. Allow both $x < 0, x > \frac{10}{3}$. Ignore any joining statements such as "and", "or" or any set notation eg. $\cup$ or $\cap$ and allow statements such as $(-\infty, 0], \left[\frac{10}{3}, \infty\right)$. $\frac{10}{3}$ does not need to be in its lowest terms. Isw after two correct inequality statements unless they contradict. (Eg ignore after two correct inequalities $x < 0, x > \frac{10}{3}$ a statement such as $0 > x > \frac{10}{3}$ but withhold the final mark if they subsequently write $0 < x < \frac{10}{3}$)

Question Number	Scheme	Marks
2 (a)	States or uses $r=1.02$	B1
	Attempts $25000\times1.02^{13}=32340$ or 32341 or 32300	M1 A1
		(3)
(b)	Attempts $\dfrac{a\left(r^n-1\right)}{\left(r-1\right)}=\dfrac{25000\left(1.02^{14}-1\right)}{1.02-1}$ or $\dfrac{125000\left(1.02^{14}-1\right)}{1.02-1}$	M1 A1
	£1 997 000	A1
		(3) **(6 marks)**

(a)

B1 States or uses $r=1.02,\ 102\%,\ (1+2\%)$ oe

M1 Attempts $25000\times"r"^{12}$ or $25000\times"r"^{13}$. Allow eg $r=2$ for this mark. They may show the increase each year which is acceptable. Expect to see at least one correct increase for their *r* and attempting 12 or 13 increases by the same percentage. Allow a misread if 2500 or 250000 is used instead.
Awrt 31706 which is 25000×1.02^{12} is usually sufficient evidence for this mark.

A1 32340 (or 32341). Must be an integer. Allow 32300 which is the answer rounded to 3sf.

(b)

M1 Attempts $\dfrac{a(r^n-1)}{r-1}$ or $\dfrac{a(1-r^n)}{1-r}$ with $a=25000$ or 125000, $n=13$ or 14 and $r=1.02$ or their *r* from part (a).

A1 For $\dfrac{25000(1.02^{14}-1)}{1.02-1}$ or $\dfrac{125000(1.02^{14}-1)}{1.02-1}$ or equivalent. Awrt 1 997 000 implies this mark.

A1 1 997 000 only

………..

Alt(b)

M1 If they attempt to add year by year then look for 13 or 14 values being added with the first year 25000 and at least one increase year to year of 2% or their *r*.

A1 All 14 values seen or implied from either column 2 or 3 of the table on the next page. Allow values to be truncated or rounded to 3 significant figures. Awrt 1 997 000 also implies this mark.

A1 1 997 000 only

n	x1.02^(n-1)	x5
1	25000	125000
2	25500	127500
3	26010	130050
4	26530	132651
5	27061	135304
6	27602	138010
7	28154	140770
8	28717	143586
9	29291	146457
10	29877	149387
11	30475	152374
12	31084	155422
13	31706	158530
14	32340	161701
	399348	1996742

Question Number	Scheme	Marks
3. (a)	$\left(1+\frac{x}{4}\right)^{12}=1+12\left(\frac{x}{4}\right)^{1}+\frac{12\times 11}{2}\left(\frac{x}{4}\right)^{2}+\frac{12\times 11\times 10}{3!}\left(\frac{x}{4}\right)^{3}+...$	M1
	$=1+3x,+\frac{33}{8}x^{2}+\frac{55}{16}x^{3}$	B1, A1
		(3)
(b)	$\left(\frac{x^2+8}{x^5}\right)\left(1+\frac{x}{4}\right)^{12}=$	
	Sight of a term independent of x = $"\frac{55}{16}"$ or $8\times {}^{12}C_5\left(\frac{1}{4}\right)^{5}$ $(=\frac{99}{16})$	B1ft,
	$"\frac{55}{16}"+8\times {}^{12}C_5\left(\frac{1}{4}\right)^{5}=\frac{55}{16}+\frac{99}{16}=\frac{77}{8}$	M1 A1
		(3) **(6 marks)**

(a)

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 3 or 4.

Accept sight of ${}^{12}C_2\left(\frac{x}{4}\right)^{2}$ or ${}^{12}C_3\left(\frac{x}{4}\right)^{3}$ condoning the omission of brackets. Accept any relevant coefficient appearing from Pascal's triangle. FYI ${}^{12}C_2=66$, ${}^{12}C_3=220$

B1 For $1+3x$ Must be simplified. Also allow $1,3x$ if written as a list.

A1 For $+\frac{33}{8}x^{2}+\frac{55}{16}x^{3}$ Accept $+4.125x^{2}+3.4375x^{3}$

(All four terms do not need to be written as a summation and may be written as a list).

(b)

B1ft Sight of $"\frac{55}{16}"$ or $8\times {}^{12}C_5\left(\frac{1}{4}\right)^{5}$ $(=\frac{99}{16})$ which must be independent of x

Most candidates do not find the additional term in the expansion of $\left(1+\frac{x}{4}\right)^{12}$ so typically would only score a maximum of 100 in (b)

M1 For attempting to add their $"\frac{55}{16}"$ to $8\times {}^{12}C_5\left(\frac{1}{4}\right)^{5}$ or $8\times 792\left(\frac{1}{4}\right)^{5}$

A1 For $\frac{77}{8}$ oe eg 9.625

Question Number	Scheme	Marks
4 (a)	-35	B1 **(1)**
(b)	Attempts $\mathrm{f}\left(\pm\frac{2}{3}\right)=0 \rightarrow \left(\frac{2}{3}-3\right)\left(3\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)+a\right)-35=0$	M1
	$-\frac{7}{3}(2+a)=35 \Rightarrow a=-17$ *	A1* **(2)**
(c)	$\mathrm{f}(x)=(x-3)(3x^2+x-17)-35=3x^3-8x^2-20x+16$	B1
	$=(3x-2)(x^2-2x-8)$	M1 A1
	$=(3x-2)(x+2)(x-4)$	M1 A1 **(5)**
		(8 marks)

(a)

B1 -35 stated

(b)

M1 Attempts to set $\mathrm{f}\left(\pm\frac{2}{3}\right)=0$. Must see $\pm\frac{2}{3}$ substituted into the expression and set equal to zero. Condone bracket errors for this mark. They may multiply out the expression before substituting so allow errors in the manipulation for this mark. (FYI $3x^3+x^2+ax-9x^2-3x-3a$)
"=0" may be implied by their working.

A1* Achieves the given answer with no incorrect work (including brackets) and at least one correct intermediate line before proceeding to $a=-17$. "=0" must be seen somewhere for this mark.
Eg. $-\frac{7}{3}(2+a)=35 \Rightarrow a=-17$

……………………………………………………………………………………………………

Alt (b)

M1 Assumes $a=-17$ and attempts $\mathrm{f}\left(\pm\frac{2}{3}\right)=0$ where $\mathrm{f}(x)=(x-3)(3x^2+x-17)-35$

A1* Achieves $\mathrm{f}\left(\frac{2}{3}\right)=0$ with no incorrect work, at least one correct intermediate line and states hence proven or some other acknowledgement of the "proof".

……………………………………………………………………………………………………..

(c)

B1 Multiplies out to achieve $\mathrm{f}(x)=3x^3-8x^2-20x+16$. This may be implied by eg $(3x-2)(x^2-2x-8)$

M1 Attempts to divide or factorise out $(3x-2)$ from their cubic.

By factorisation look for $3x^3 \pm ...x^2 \pm ...x \pm 2\alpha = (3x-2)(x^2+kx\pm\alpha)$ $k \neq 0$ or they may equate coefficients correctly to find *k*.

By division look for
$$3x-2\overset{\displaystyle x^2 \pm \frac{(\beta+2)}{3}x.........}{\overline{\big)\,3x^3 \pm \beta x^2..............}}$$
$$\underline{3x^3-2x^2}$$
(Usually $x^2 \pm 6x....$)

There are various other methods but we should be seeing a correct method to find at least two of the coefficients (allow ±) of their quadratic for the method mark. Send to review if unsure how to mark a response.

A1 Correct factors $(3x-2)(x^2-2x-8)$ The two factors do not need to be written together eg this mark can be awarded if the quadratic factor is seen at the top of their algebraic division.

M1 Factorises their quadratic factor of the form Ax^2+Bx+C with *A*, *B*, *C* $\neq 0$. Score for $(Ax^2+Bx+C)=(dx+e)(fx+g)$ where $|A|=|d\times f|$ or $|C|=|e\times g|$

A1 $(3x-2)(x+2)(x-4)$ on one line following B1M1A1M1

isw $(3x-2)(x+2)(x-4) \Rightarrow x=\frac{2}{3},-2,4$ and allow $3(x-\frac{2}{3})(x+2)(x-4)$

Note the question says using algebra and showing each step.

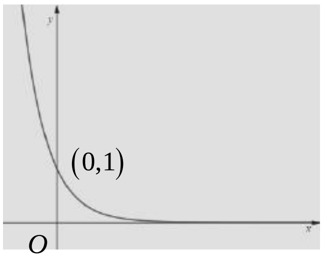
$\mathrm{f}(x)=3x^3-8x^2-20x+16 \Rightarrow (3x-2)(x+2)(x-4)$ is 10000.

Beware $\mathrm{f}(x)=3x^3-8x^2-20x+16=0 \Rightarrow x=\frac{2}{3},-2,4 \Rightarrow (3x-2)(x+2)(x-4)$ is 10000

$\mathrm{f}(x)=3x^3-8x^2-20x+16=0 \Rightarrow (x-\frac{2}{3})(x+2)(x-4)$ is 10000

isw $(3x-2)(x+2)(x-4) \Rightarrow x=\frac{2}{3},-2,4$

isw $3(x-\frac{2}{3})(x+2)(x-4) \Rightarrow (x-\frac{2}{3})(x+2)(x-4)$

Question Number	Scheme	Marks
5.(a)	 Shape or intercept at 1	M1
	Fully correct	A1
		(2)
	$h=0.5$	B1
(b)	Area $\approx \frac{0.5}{2}\{4.25+16.06+2\times(6.427+9.125+12.34)\}$	M1
	= awrt 19.0	A1
		(3)
(c)	$\int_2^4 \left(x(x-3)+\left(\frac{1}{2}\right)^x\right)dx = \int_2^4 \left(x^2+\left(\frac{1}{2}\right)^x-3x\right)dx = (b)-\left[\frac{3}{2}x^2\right]_2^4$	M1
	= awrt 1.0	A1ft
		(2)
		(7 marks)

(a)

M1 For either the shape in quadrants 1 and 2 only or the y-intercept at 1. Condone slips of the pen but the gradient should be tending to 0 as x increases. Condone (1,0) marked on the correct axis.

A1 Fully correct. Shape in quadrants 1 and 2 only with the y-intercept at 1. Condone slips of the pen towards the x-axis but the graph should not cross the x-axis. Condone (1,0) marked on the correct axis. For a fully correct graph its asymptote should not appear to be $y=1$. As a rule of thumb look for the asymptote to be at least half way below its y-intercept.

Do not be too concerned about relative position of the curve in the second quadrant as long as the curve gets steeper as $x\to-\infty$

(b)

B1 For $h=0.5$ seen or implied by sight of $\frac{0.5}{2}$ in front of the bracket

M1 For the correct bracket structure $\{4.25+16.06+2\times(6.427+9.125+12.34)\}$ condoning slips copying from the table or the omission of the final bracket.

eg $\frac{1}{2}\times0.5\times(4.25+16.06+2\times(6.427+9.125+12.34)$

Recovery from missing brackets to achieving the correct answer can still score full marks.

A1 19.0235 but awrt 19.0 will usually score full marks

Note: The calculator answer for the integral is 18.937… which is A0.

Examples of mark traits where full marks are not scored:

$\frac{1}{2}\times 0.5\times 4.25+16.06+2\times(6.427+9.125+12.34)\quad (=72.9065)$ is B1M0A0

$\frac{1}{2}\times 0.5\times 4.25+16.06+2\times 6.427+9.125+12.34\quad (=51.4415)$ is B1M0A0

(c)

M1 For realising that $\int_2^4\left(x(x-3)+\left(\frac{1}{2}\right)^x\right)\,dx=\int_2^4\left(x^2+\left(\frac{1}{2}\right)^x-3x\right)\,dx=(b)-\left[\frac{3}{2}x^2\right]_2^4$

i.e. $(b)-\left[\frac{3}{2}x^2\right]_2^4$ must be seen or $(b)-\int_2^4(3x)\,dx\Rightarrow$ (b) -18 for this mark. The $3x$ term must be isolated from the rest of the integral that is identified in some way to be their part (b). **There must be some integration carried out** so look for $3x\rightarrow ...x^2$. Condone poor notation.

A1ft 1.0235 but accept awrt 1.0 or follow through on their answer or their unrounded answer from (b) -18 **provided** M1 has been scored.

Note: The calculator answer for the integral is 0.937 (awrt 0.9) is M0A0

Question Number	Scheme	Marks
6(a)	Attempts line with gradient -2 and point $(4,-1)$	
	$y+1=-2(x-4)$	M1
	$y=-2x+7$	A1
		(2)
(b)	$y=\frac{1}{2}x$ meets $y=-2x+7$ when $\frac{1}{2}x=-2x+7 \Rightarrow x=\frac{14}{5}, y=\frac{7}{5}$ oe	M1 A1
	Attempts $r^2=\left(4-"\frac{14}{5}"\right)^2+\left(-1-"\frac{7}{5}"\right)^2=\frac{36}{5}$ oe	dM1 A1
	$(x-4)^2+(y+1)^2=\frac{36}{5}$ oe	A1
		(5)
		(7 marks)

(a)

M1 Attempts the equation of the line with gradient -2 and point $(4,-1)$
Condone a slip on one of the signs of the $(4,-1)$. For example $y-1=-2(x-4)$
If the form $y=mx+c$ is used they must proceed as far as $c=...$

A1 $y=-2x+7$

………

Alt (a)

M1 Differentiates implicitly the equation of the circle $(x-4)^2+(y+1)^2=r^2 \Rightarrow 2(x-4)+2(y+1)\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and substitutes in $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$

A1 $y=-2x+7$

………

(b)

M1 Attempts to find where $y=\frac{1}{2}x$ meets their $y=-2x+7$. Expect to see candidates proceed to $x=...$

A1 $\left(\frac{14}{5},\frac{7}{5}\right)$ or any other form eg $(2.8,1.4)$ or $x=\frac{14}{5}, y=\frac{7}{5}$

dM1 Attempts to find the distance between their $\left(\frac{14}{5},\frac{7}{5}\right)$ and $(4,-1)$ using Pythagoras' theorem. Look for an attempt to subtract the coordinates, square and add. It is dependent on the previous method mark.
Condone a slip on one of the signs.

A1 For $(r=)\sqrt{\frac{36}{5}}$ or $(r^2=)\ \frac{36}{5}$ oe

A1 $(x-4)^2+(y+1)^2=\frac{36}{5}$ or $(x-4)^2+(y+1)^2=7.2$ or $5(x-4)^2+5(y+1)^2=36$ only

………

Alt (b)

M1 Substitutes $y=\frac{1}{2}x$ into $(x-4)^2+(y+1)^2=r^2$, multiplies out and rearranges $\ldots=0$

A1 $5x^2-28x+68-4r^2=0$ or $5y^2-14y+17-r^2$

dM1 Uses the discriminant $b^2-4ac=0$ and proceeds to $r=\ldots$ or $r^2=\ldots$ It is dependent on the previous method mark.

A1 For $r=\sqrt{\frac{36}{5}}$ or $r^2=\frac{36}{5}$

A1 $(x-4)^2+(y+1)^2=\frac{36}{5}$ oe eg $(x-4)^2+(y+1)^2=7.2$ or $5(x-4)^2+5(y+1)^2=36$

Note: Candidates who substitute $y=-2x+"7"$ into $(x-4)^2+(y+1)^2=r^2$ and attempt the discriminant approach will score 0 marks in (b)

Question Number	Scheme	Marks
7. (i)	$\log_a\left(\frac{\sqrt{a}}{b}\right)=\frac{1}{2}\log_a a-\log_a b=\frac{1}{2}-k$	M1 A1
		(2)
(ii)	$\frac{\log_a a^2b}{\log_a b^3}=\frac{2\log_a a+\log_a b}{3\log_a b}=\frac{2+k}{3k}$	M1 A1
		(2)
(iii)	$\sum_{n=1}^{50}\left(k+\log_a b^n\right)=50k+\left(1k+2k+3k+...+50k\right)$ or $\left(2k+3k+4k+...+51k\right)$	
	Uses the sum formula an AP with $n=50, d=k$	M1
	$S=50k+\frac{50}{2}\left(2k+49k\right)$ $\quad$ $S=\frac{50}{2}\left(2k+51k\right)$	A1
	$=1325k$	A1
		(3) **(7 marks)**

(i)

M1 Uses log laws to achieve $\frac{1}{2}\log a-\log b$ or eg $0.5\log a-\log b$

A1 $\frac{1}{2}-k$ oe

(ii)

M1 Uses correct log laws on the numerator or denominator. Score for $\frac{...}{3\log b}$ or $\frac{...}{3k}$ or sight of the numerator $2\log a+\log b$ which does not have to be part of a fraction (May be implied by $2+k$)

A1 $\frac{2+k}{3k}$ or $\frac{2}{3k}+\frac{1}{3}$ do not isw

(iii)

M1 **Either** uses the sum formula for an AP in terms of k on part or all of the expression with $n=50, d=k, a=k$ or $n=50, d=k, a=2k$. It is sufficient to see the terms substituted in for this mark.

$\sum_{n=1}^{50}nk=\frac{50}{2}\left(2k+49k\right)$ or $\sum_{n=1}^{50}(k+nk)=\frac{50}{2}(2k+51k)$.

You may see the equivalent sum $\frac{n}{2}(a+L)$ eg $\frac{50}{2}(k+50k)$

Or uses the sum formula for an AP in terms of $\log_a b$ on part of the expression with $n=50, d=\log b, a=\log b$

$\sum_{n=1}^{50}\log b^n=\frac{50}{2}\left(2\log b+49\log b\right)$

You may see the equivalent sum $\frac{n}{2}(a+L)=\sum_{n=1}^{50}\log b^n=\frac{50}{2}\left(\log b+50\log b\right)$

A1 A correct unsimplified answer in terms of k.

Eg. $S = 50k + \frac{50}{2}(2k + 49k)$ or $S = \frac{50}{2}(2k + 51k)$

A1 $1325k$

Question Number	Scheme	Marks
8 (i)	$\int \frac{8\sqrt{x}-5}{2x^2}\,dx = \int 4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2}\,dx$	B1
	$= -8x^{-\frac{1}{2}} + \frac{5}{2}x^{-1} \quad (+C)$	M1 A1
	$\int_2^4 \left(4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2}\right)dx = \left(-4+\frac{5}{8}\right) - \left(-4\sqrt{2}+\frac{5}{4}\right) = 4\sqrt{2} - \frac{37}{8}$	dM1 A1
		(5)
(ii)	$\int \left(\frac{1}{2}x^2 + k\right)dx = \left[\frac{1}{6}x^3 + kx\right]$	M1 A1
	$\int_{-3}^{6}\left(\frac{1}{2}x^2 + k\right)dx = 55 \Rightarrow \left[\frac{1}{6}x^3 + kx\right]_{-3}^{6} = 55 \Rightarrow (36+6k) - \left(-\frac{9}{2} - 3k\right) = 55 \Rightarrow k = ...$	dM1
	$k = \frac{29}{18}$	A1
		(4) **(9 marks)**

(i)

B1 For $\frac{8\sqrt{x}-5}{2x^2} \to 4x^{-\frac{3}{2}} - \frac{5}{2}x^{-2}$ (seen or implied). Ignore any $+C$

You may also see a factor of ½ taken out eg $\frac{8\sqrt{x}-5}{2x^2} \to \frac{1}{2}\left(8x^{-\frac{3}{2}} - 5x^{-2}\right)$

M1 For raising any index by one on one of their terms. Follow through on incorrect indices but do not allow for candidates who attempt to integrate the numerator and denominators separately.

A1 Correct terms (may be unsimplified but the indices must be processed)

dM1 Substitutes 4 and 2 into an integrated function and subtracts either way round. Dependent on the previous method mark. awrt 1.03 following B1M1A1 is sufficient evidence to award this mark.

A1 $4\sqrt{2} - \frac{37}{8}$ or exact equivalent (eg $4\sqrt{2} - 4.625$ or $\frac{32\sqrt{2}-37}{8}$)

Note: Answer only scores 0 marks.

(ii)

M1 Attempts to integrate $\left(\frac{1}{2}x^2 + k\right)$ with one term correct unsimplified so allow eg $\frac{\frac{1}{2}x^3}{3}$ or kx^1 but the index must be processed.

A1 $\frac{1}{6}x^3 + kx$ (allow unsimplified equivalent expressions as above)

dM1 Substitutes 6 and -3 into $Ax^3 + kx$, subtracts, sets $= 55$ and proceeds to $k = \ldots$. Do not be concerned with the mechanics of their rearrangement.

A1 $k = \frac{29}{18}$ or exact equivalent eg $1.6\dot{1}$ but do not allow 1.6 or 1.61 or 1.61….

Question Number	Scheme	Marks
9.(i)	$3\sin(2\theta-10°)=1\Rightarrow(2\theta-10°)=\arcsin\left(\frac{1}{3}\right)$	M1
	$\theta=\frac{19.47+10}{2},\frac{160.53+10}{2}$	dM1
	$\theta=\text{awrt }14.7°, 85.3°$	A1, A1
		(4)
(ii) (a)	Writes $\frac{1}{\tan\alpha}-\sin\alpha=2\sin\alpha-\frac{1}{\tan\alpha}$ oe	M1
	$\frac{2}{\tan\alpha}=3\sin\alpha\Rightarrow\frac{2\cos\alpha}{\sin\alpha}=3\sin\alpha\Rightarrow 2\cos\alpha=3\sin^2\alpha$ *	dM1 A1*
		(3)
(b)	$2\cos\alpha=3\sin^2\alpha\Rightarrow 2\cos\alpha=3\left(1-\cos^2\alpha\right)$	M1
	$3\cos^2\alpha+2\cos\alpha-3=0$	A1
	Attempts to solve $3\cos^2\alpha+2\cos\alpha-3=0\Rightarrow\cos\alpha=\frac{-2\pm\sqrt{40}}{6}$ oe	dM1 A1
	$\alpha=5.517$ radians	A1
		(5)
		(12 marks)

(i)

M1 For proceeding to $x=\arcsin\left(\frac{1}{3}\right)$ which may be implied by the sight of awrt $19.5°$ or awrt $160.5°$ (Allow awrt 0.340 (radians) for this mark)

dM1 For correct order of operations leading to one answer for θ. May be implied by $14.7°/14.8°$ but cannot be scored by adding 10 to an angle in radians but may be implied by awrt $0.257/0.258$ rad

A1 One of $\theta=\text{awrt }14.7°,85.3°$ ignore any others.

A1 Both of $\theta=\text{awrt }14.7°,85.3°$ and no others in the range.

Note that solutions based entirely on graphical or numerical methods are not acceptable so answers only will score 0 marks.

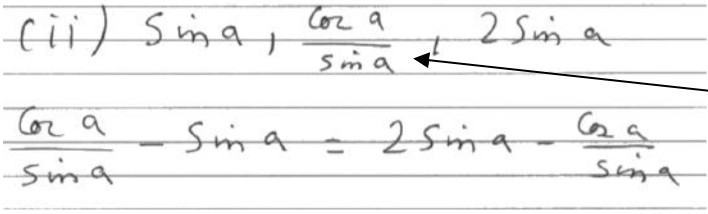
(ii)(a)

M1 Uses the terms of an AP to set up a correct equation. Eg $\frac{1}{\tan\alpha}-\sin\alpha=2\sin\alpha-\frac{1}{\tan\alpha}$, $\frac{\cos\alpha}{\sin\alpha}-\sin\alpha=2\sin\alpha-\frac{\cos\alpha}{\sin\alpha}$. They may also do $2\left(\frac{1}{\tan\alpha}-\sin\alpha\right)=2\sin\alpha-\sin\alpha$. Condone mixed variables and poor notation for the method marks.

dM1 Uses $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ within an equation involving $\dfrac{1}{\tan\alpha}, \sin\alpha$ & $2\sin\alpha$. This mark can be implied but do not award if they just proceed straight to the final answer. It is dependent on the previous method mark. A candidate starting with $\dfrac{\cos\alpha}{\sin\alpha} - \sin\alpha = 2\sin\alpha - \dfrac{\cos\alpha}{\sin\alpha}$ scores M1dM1 straight away.

A1* Proceeds to given answer with no errors or omissions. The equation must start with an equation involving $\tan\alpha$ but see the note below for further guidance. Withold this mark for incorrect notation eg $\sin\alpha^2$ or if they had mixed variables on the same line. Eg α and θ.

Note: In the example below they can score full marks as $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ is implied by their second term listed on the first line. Had they not written these first three terms or stated somewhere in their solution $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ in their working then we would withhold the final mark.



$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ implied here

(ii)(b)

M1 Attempts to use $\sin^2\alpha + \cos^2\alpha = 1$ Eg. $2\cos\alpha = 3\sin^2\alpha \Rightarrow 2\cos\alpha = 3(\pm1 \pm \cos^2\alpha)$. Beware that a candidate may use this identity in part (ii)(a) which would not gain the mark in this part.

A1 $3\cos^2\alpha + 2\cos\alpha - 3 = 0$ The "=0" may be implied by later work but the terms must be collected on one side. Evidence of this may be awarded for correct coefficients substituted into the quadratic formula.

dM1 Attempts to solve their $3\cos^2\alpha + 2\cos\alpha - 3 = 0$ by the formula/completing the square, usual rules for solving a 3TQ apply but do not award for attempted factorisation unless their quadratic factorises. Award for $(\cos\alpha =)\ \dfrac{-1 \pm \sqrt{10}}{3}$ or $(\cos\alpha =)$ awrt 0.72 or awrt -1.4 or equivalent. You may need to check decimal values on your calculator.

A1 $(\cos\alpha =)\ \dfrac{-2+\sqrt{40}}{6}$ oe (typically $(\cos\alpha =)\ \dfrac{-1+\sqrt{10}}{3}$). Implied by $(\cos\alpha =)$ awrt 0.721or $(\alpha =)$ awrt 5.52 radians.

A1 $(\alpha =)$ awrt 5.517 radians and no others in the given range

Question Number	Scheme	Marks
10.(a)	$x=2, y=5 \Rightarrow 5=8a-12+6+b$	M1
	$\frac{dy}{dx}=3ax^2-6x+3$ **AND** $x=2, \frac{dy}{dx}=7 \Rightarrow 7=12a-12+3$	M1
	Solves $11=8a+b$ and $7=12a-9 \Rightarrow a=\frac{4}{3}, b=\frac{1}{3}$	A1 A1
		(4)
(b)	Sets $\frac{dy}{dx}=3ax^2-6x+3=0$ with their value of a and b	M1
	$4x^2-6x+3=0$ and attempts "b^2-4ac"	dM1
	$b^2-4ac=-12<0$ hence there are no turning points oe	A1*
		(3) **(7 marks)**

(a)

M1 Substitutes $x=2, y=5$ into $y=ax^3-3x^2+3x+b$ to get an equation in a and b (condone slips)

M1 Substitutes $x=2, \frac{dy}{dx}=7$ into $\frac{dy}{dx}=3ax^2-6x+3$ to get an equation in a ($\frac{dy}{dx}$ must be correct)

A1 $a=\frac{4}{3}$ or exact equivalent

A1 $b=\frac{1}{3}$ or exact equivalent

(b)

M1 Sets their $\frac{dy}{dx}=0$ with their value of a. This may be implied by later working.

dM1 Attempts to find b^2-4ac or roots via the formula

A1* Achieves $4x^2-6x+3=0$, $b^2-4ac=-12$ and states $-12<0$ and so there are ***no turning points*** or equivalent. They may attempt to solve the equation and either state that ***no real roots so no turning points*** or achieve complex roots and state that ***no real roots so no turning points***. Note full marks can be scored with an incorrect value for $b=\frac{1}{3}$ but cannot be scored from an incorrect value for $a=\frac{4}{3}$ from part (a)

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Alt (b)

M1 Attempts to complete the square for $\frac{dy}{dx}=3ax^2-6x+3$ for their value of a to achieve eg $4(x\pm...)^2$ or $(2x\pm...)^2$

dM1 $4x^2-6x+3=4\left(x\pm\frac{3}{4}\right)^2\pm...$ or $4x^2-6x+3=\left(2x\pm\frac{3}{2}\right)^2\pm...$

A1* Achieves $4x^2-6x+3=4\left(x-\frac{3}{4}\right)^2+\frac{3}{4}$ or $4x^2-6x+3=\left(2x-\frac{3}{2}\right)^2+\frac{3}{4}$ and states there are ***no turning points*** as $\frac{dy}{dx}>0$ (for all x) or equivalent.

If you see any other ways that may be credit worthy then send to review